

Standard Grand Unification from superstrings

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arXiv:hep-th/9507162v1 28 Jul 1995

Abstract

Recent developments [1, 2] about the construction of standard $SO(10)$ and $SU(5)$ grand unified theories from 4-dimensional superstrings are presented. Explicit techniques involving higher level affine Lie algebras, for obtaining such stringGUTs from symmetric orbifolds are discussed. Special emphasis is put on the different constraints and selection rules for model building in this string framework, trying to disentangle those which are generic from those depending on the orbifold construction proposed. Some phenomenological implications from such constraints are briefly discussed.

Talk at Susy 95, Paris, May 1995

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1 Introduction

The motivations for considering the construction of GUTs in the framework of string theory are manifold and rely on different basis or beliefs. The main reason is undoubtedly the striking fitting of $\sin^2\theta_W$ vs. α_s when both supersymmetry and unification are assumed [3]. Many of the virtues emphasised by the first GUTs builders more than a decade ago, are still appealing today. The fitting of SM content into simple GUT multiplets (predicting that right handed quarks are $SU(2)$ singlets etc.) is an example. Therefore it seems worth exploring the possibility of having GUTs like models in the context of a frame theory for unifying gauge and gravitational interactions. We will call this type of models StringGUTs [1].

Let us mention, however, that some other features signaled in GUTs days appear less compelling in the presence of string theory. Charge quantization for example may be understood in this context from anomaly cancellation. Indeed, even the attractive idea of coupling constants unification could perhaps be achieved at the string scale directly from Standard Model. Introduction of non fully controllable features would be required in this case (see Ref.[4] for a discussion of this possibility). Nevertheless, the main obstacle that remains is the lack of a doublet-triplet splitting mechanism (unless anthropic principle is called for fine tuning) for avoiding fast proton decay. If StringGUTs are meaningful, they should furnish at least a possible hint for solving this problem.

In building up StringGUTs we will not only be interested in having GUT groups like $SU(5)$ or $SO(10)$, but also in their matter content (Higgses, fermion generations) and in the allowed couplings indicating possible symmetry breaking patterns, mass generating terms, doublet triplet splitting terms etc., present at GUT scale.

The gauge group G information in string theory relies on the presence of an affine Lie algebra at level k , realized by currents $J_a(z)$ ($a = 1, \dots, \text{rank}G$) living on the world sheet of the string. Namely they satisfy

$$J_a(z)J_b(w) = \frac{k\delta_{ab}}{(z-w)^2} + \frac{if_{abc}J^c}{(z-w)} + \dots \quad (1)$$

where f_{abc} are the structure constants of the group Lie algebra. Whereas in ten dimensions the only consistent groups are $E_8 \times E_8$ or $SO(32)$ realized at level $k = 1$, in four dimensions the situation is much less constrained. The gauge group generically appears as a product of non-abelian gauge groups G_i , realized at levels k_i , $G_1 \times G_2 \times \dots$ times $U(1)$ factors. However, the values of levels are not completely arbitrary. In fact, vanishing of conformal anomaly requires $k \leq 7$ for $SO(10)$, $k \leq 4$ for E_6 or the more relaxed condition $k \leq 55$ for $SU(5)$. These are upper bounds; actual computations in specific models prove to be much more restricted. The methods proposed below, for example, cannot produce levels beyond $k = 3$ (and probably not bigger than $k = 2$) for the GUT groups we are interested in. In particular, most of the models considered in the literature are normally obtained at level 1.

The values attained by level k are very important, since they impose (unitarity) limits on possible representations allowed. In particular for level 1, only vector or spinor representations are admitted in $SO(2N)$ and only the fundamental or 10 (and their conjugates) for $SU(5)$ group.

In supersymmetric GUTs, it is crucial to have quiral fields in the adjoint (or bigger representations) in order to achieve the breaking of the GUT symmetry down to the Standard Model. In the context of $N = 1$ four dimensional strings, this requires GUT groups realized

SU(5)	5	10	24	15	40	50
k = 1	2/5	3/5	-	-	-	-
k = 2	12/35	18/35	5/7	28/35	33/35	-
SO(10)	10	16	45	54	120	126
k = 1	1/2	5/8	-	-	-	-
k = 2	9/20	9/16	4/5	1	-	-

Table 1: Conformal weights h_{KM} for different representations of the unifying groups $SU(5)$ and $SO(10)$.

at levels $k \geq 2$. Therefore, building higher level gauge groups becomes a necessary task. This problem was addressed in orbifold models in Ref.[5]. In Ref.[6] it was considered for fermionic string constructions and recent GUT model building attempts in this context may be found in [7].

In our specific work we will be mainly concerned with symmetric orbifolds (see [1, 2] for a more detailed list of references.) This type of construction is not better, in principle, than other four dimensional strings pictures. The main advantage is, perhaps, that models are easy to handle, in the sense that many desired aspects can be controlled without need of computers. Furthermore, many consistency checks are available and world-sheet supersymmetry (which may be a problem in fermionic constructions [8]) is ensured from the beginning.

Before getting into these orbifold models, let us first briefly discuss another stringent stringy constraint for low energy model building. When considering effective theories, only massless string states will be relevant. The nature of this constraint relies on the conformal structure of the theory and may be stated in simple words by saying that quantum numbers do weigh (conformally). Thus, the more quantum numbers a particle has, the less likely it is for it to be in the massless spectrum. This is summarized in the following mass formula

$$\frac{1}{8}M_L^2 = h_{KM} + h_{int} - 1 . \quad (2)$$

where h_{int} is the contribution to the mass of the particle from the internal (compactified) sector and h_{KM} is the contribution of the gauge sector to the conformal weight of the particle. For a state in a representation R of a given non abelian gauge group it is given by

$$h_{KM} = \sum_i \frac{C(R_i)}{k_i + \rho_i} \quad (3)$$

where $C(R)$ is the quadratic Casimir of the representation R , thus growing with the size of R , illustrating what we said above. The allowed massless representations must then satisfy $h_{KM} \leq 1$. For GUT groups $SO(10)$ and $SU(5)$, at levels $k = 1, 2$ they are given in Table 1. Therefore, some relevant phenomenological information may be already extracted from very general considerations. For example, a missing partner mechanism (which needs $50 - plet$) is not allowed for in $SU(5)$, a 126 in $SO(10)$ cannot be used for producing fermion mass matrices or see-saw mechanism etc.. It is also worth noticing that the 54 representation in $SO(10)$ is rather peculiar. Its conformal weight being $h_{54} = 1$ tells us that it cannot be charged with respect to other gauge groups and that the internal right handed part of its vertex operator is trivial. Couplings with other states will therefore be quite restricted. Moreover, a 54 can only

live in a sector with vanishing internal energy. For symmetric orbifolds this sector must be one (an order two) of the untwisted sectors and consequently, from orbifold selection rules, it can be shown that self couplings are not allowed for. These are indications (which can be made more explicit in specific constructions) that a 54 behaves like a string modulus [1].

The starting point in our explicit StringGUTs construction is (0,2) symmetric orbifold of a four dimensional heterotic string. It proves convenient in this case to deal with $Spin(32)$ instead of $E_8 \times E_8$ gauge lattice. A replicated gauge group structure $G_{GUT} \otimes G'$ with $G_{GUT} \in G'$ is then looked for by an adequate embedding of the orbifold action (as a shift vector, a lattice automorphism etc.) into the gauge degrees of freedom. The corresponding generators and levels are $(J, k = 1)$ and $(J', k' = 1)$. The second step in our construction relies on the introduction of a projection selecting only diagonal $J_a + J'_a$ combinations. Thus, from eq. (1) and by noticing that generators of the replicated groups commute among themselves ($JJ' = 0$), we see that the diagonal group G_{GUT}^D emerges at level $k + k' = 2$ as desired. Of course, the introduction of such a projector implies the simultaneous appearance of twisted sectors (and related constraints), in order to maintain modular invariance (see Ref.[2] for a detailed discussion of constraints).

Notice that when the level 2 diagonal group is selected, a coset structure emerges, contributing with the missing conformal structure. Consider the interesting case of $SO(10) \times SO(10)$ with central charge $c = 10$ broken to the diagonal group $SO(10)_2$ at level 2, with central charge $c_2 = 9$. The missing unity of central charge is provided by the coset $\frac{SO(10) \times SO(10)}{SO(10)_2}$. This is a rather peculiar coset. Its charge being one, it must be equivalent to a free (orbifoldize) compactified S_1 boson [9]. In fact it corresponds to S_1/Z_2 at the compactifying radius $\sqrt{\frac{5}{2}}$.

Let us stress that the general basics of the methods discussed above, rather than being exclusive of orbifolds, are quite general. They could be implemented, for example, in $N = 2$ coset constructions like Gepner [10] or Kazama-Suzuki [11] models, by embedding an order two internal modding as a permutation of the replicated GUT groups. The replicated structure could be obtained by embedding a shift into the gauge lattice as explained in Ref. [12]. In this case, $SO(26)$ rather than $SO(10) \times E_8$ might be probably preferred.

The breaking to G_{GUT}^D in orbifold models may be achieved through the methods proposed in Ref. [5]. Method (I) exploits the fact that, when an orbifold twist is embedded into the gauge degrees as an automorphism of the gauge lattice- in this present case used to obtain the structure $G_{GUT} \otimes G'$ - non quantized Wilson line are allowed for. They are then subsequently added in order to continuously break to level 2, diagonal GUT group. In the second method (II) the replicated structure is usually achieved through a shift and last breaking is realized by modding by a Z_2 which permutes G_{GUT} and G'_{GUT} inside G' . The third method is field theoretical. The diagonal group is obtained by looking for flat directions of the effective superpotential. Let us mention that even if these procedures look quite different, in many cases the same model at $k = 2$ is obtained. Nevertheless, method (II) is in some aspects more versatile. For instance the permutation twist may be accompanied by a discrete shift which may be used to pick up a 45-plet (it never appears in method (I) or (II)) of $SO(10)$ instead of a 54.

More explicit information may be obtained in the symmetric orbifold context from eq.(2). In fact, now the internal energy can be explicitly computed in terms of the vacuum energy $E_0 = \sum_{i=1}^3 \frac{1}{2}|v_i|(1 - |v_i|)$ where v_i ($i = 1, 2, 3$) are known twist eigenvalues for each twisted sector. We learn that all level 2 representations in Table 1 are allowed in the untwisted sectors. In twisted sectors a 45 could only fit in sectors with $v = 1/4(0, 1, 1)$ or $v = 1/6(0, 1, 1)$ (further analysis discards even this possibility), 24-plets of $SU(5)$ are forbidden in sectors Z_3 , Z_4 , Z'_6 and Z_8 orbifolds.

<i>Sector</i>	$SO(10) \times SO(8)$	Q	Q_A
U_1	(1,8)	1/2	1/2
	(1,8)	-1/2	-1/2
U_2	(1,8)	-1/2	1/2
	(1,8)	1/2	-1/2
U_3	(54,1)	0	0
	(1,1)	0	0
	(1,1)	0	1
	(1,1)	1	0
	(1,1)	-1	0
	(1,1)	0	-1
θ	$3(16, 1)$	1/4	1/4
	$(\overline{16}, 1)$	-1/4	-1/4
ω	$3(16, 1)$	-1/4	1/4
	$(\overline{16}, 1)$	1/4	-1/4
$\theta\omega$	$4(10, 1)$	0	1/2
	$4(10, 1)$	0	-1/2
	$3(1, 8)$	0	1/2
	(1, 8)	0	-1/2
	$8(1, 1)$	1/2	0
	$8(1, 1)$	-1/2	0

Table 2: Particle content and charges.

As an illustration of what we have been discussing above, consider the following $SO(10)_2 \times SO(8) \times U(1)^2$ StringGUT model in a $Z_2 \times Z_2$ symmetric orbifold. This model can be obtained from the three methods discussed above. Consider method (II). A first breaking to $SO(10) \times SO(18) \times U(1)^2$ at level one is achieved by embedding the orbifold twists θ of the first Z_2 as a shift $A = 1/2(1111100000000010)$ into the gauge degrees of freedom, and similarly for the other twist ω , as a shift $B = \frac{1}{2}(0000000000000011)$. The breaking to the diagonal group is then achieved by assigning to a second order direction of the compactifying cubic lattice, a Wilson line Π acting as a permutation of the first two blocks of five gauge bosons coordinates,

$$\Pi(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} | \dots, X_{16}) = \quad (4)$$

$$(X_6, X_7, X_8, X_9, X_{10}; X_1, X_2, X_3, X_4, X_5 | \dots, X_{16}) \quad (5)$$

This model has four generations. Its massless spectrum is presented in Table 2. Q and Q_A are the corresponding $U(1)$ charges. Q_A is anomalous, the anomaly cancelling, as usual, through the Green-Schwarz mechanism.

Let us conclude with a brief summary of the constraints and selection rules for GUT model building from superstrings. As we mentioned, some of these are very general and rely on the 4-d heterotic string framework. All terms in the effective superpotential must have $dim \geq 4$ (mass terms), only level 2 representations presented in Table 1 are admitted, $\underline{54} \ \underline{54}X$ or $\underline{5454'}X$ couplings, where X is a singlet are forbidden. Other rules depend on the symmetric orbifold construction and on the methods proposed for level 2 models construction. It can be seen for instance that: 1) There is place for only one $\underline{54}$ or alternatively one $\underline{45}$ $SO(10)$. They

must lie in an order two untwisted sector. 2) As a consequence, no self couplings $\underline{54}^n$ (or $\underline{45}$) are possible. 3) Couplings $\underline{45} \underline{45} X$ are also forbidden etc. An extensive discussion of these rules will appear in Ref. [2].

With this type of constraints, it seems quite difficult to achieve GUT symmetry breaking while keeping the MSSM particle content needed for direct coupling unification. Then we may say that in general, extra particles besides those in the MSSM spectrum will remain massless. For example if $SU(5)$ breaking is achieved through the adjoint $\underline{24}$, the partners of Goldstone bosons, which usually acquire mass through self couplings in the potential, will remain massless. They transform as $(8, 1, 0) + (1, 3, 0) + (1, 1, 0)$ under $SU(3) \times SU(2) \times U(1)$ and there is no direct coupling unification close to 10^{16}Gev with this extra matter. It remains to be seen if this is a generic, unavoidable feature, calling for the introduction of an intermediate scale, or if models with the MSSM minimal particle content may be found. Models such as those proposed in [13], (without self interactions, with no bigger than $\underline{54}$ dimensional representations..) look quite close to string GUTs building requirements. However some (from general rules) or many (in orbifold constructions) of the terms present in such superpotentials are forbidden in the string framework. Let us mention that, in spite of the severe constraints we are finding, many of the most relevant terms needed in standard GUTs do reappear here. This is quite interesting, because this was by no means guaranteed from the beginning and a completely different scenario could have emerged. For instance, in the model presented above there is a $\underline{54}$ and $\bar{\underline{16}} + \underline{\bar{16}}$ and $\underline{10}$ s, needed for GUT symmetry breaking down to the standard model. There are couplings $\underline{10} \underline{10} \underline{16}$ needed for fermion masses, $\underline{16}_i \bar{\underline{16}} X_i$ ($i = 1, \dots, N_{gen}$) with X a singlet, which could be used for neutrino masses etc. The fact that usually four generation (or multiples of four) models are found, is related to the kind of orbifolds we have been considering so far ($Z_2 \times Z_2, Z_4 \times Z_4 \dots$). Three generations would be easier to be found in orbifolds admitting order three Wilson lines ($Z_6 \dots$)[2].

The other fundamental unanswered question in SusyGUTs refers to the doublet triplet splitting problem. Interestingly enough, most of the models found posses terms of the form

$$W_X = \lambda H \Phi \bar{H} + X H \bar{H} . \quad (6)$$

where $\Phi = \underline{54}$ ($\underline{24}$) $H = \underline{10}$ ($\underline{5}$) and X a singlet of $SO(10)$ ($SU(5)$). For instance, in the the $SO(10)$ model presented above, it is possible to see that there exist flat directions which break the symmetry down to $SU(4) \times SU(2) \times SU(2)$ with some of the doublets remaining light whereas the colour triplets remain heavy. It is well known that this mechanism is spoiled by quantum corrections in field theory [14]. Studying its stability in string theory would involve non perturbative physics determining preferred directions in moduli space. Because of our present ignorance, we can not rule out this mechanism in the string context. Recall that there is no place for a missing partner mechanism (at least for $k \leq 5$) and that fine tuning is not even possible here since there are no mass terms.

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